Gravitational energy density, gravitational waves

Marijan Koželj, M.Sc., El.Eng.*

October 20, 2016

Abstract

The vacuum is the lowest energy state of the space. The energy density of electric and magnetic fields in space is generally known. The gravitational field represents the energy density which is in our sensory perception world, in comparison with the two previously mentioned, very large. The examples of calculated energy densities, of electric, magnetic, and gravitational field are shown. Changes in gravitational fields propagate as gravitational waves throughout the space. Fields are energy, they are intermediary of energy transmission through space, i.e., energy propulsion ("push off") throughout the space.

Keywords: gravitational energy, energy density in the vacuum, gravitational waves

1 The energy density in the electric and in the magnetic field

In the electric field strength $E = 1kV/mm = 1\ MV/m$, the energy density is

$$w_E = \frac{1}{2}E^2 \cdot \epsilon_0 = \frac{1}{2}(1 \cdot 10^6)^2 \cdot (8.854 \cdot 10^{-12}) = 4.43\ J/m^3\ , \tag{1}$$

$\epsilon_0 = 8.854 \cdot 10^{-12}\ As/Vm$ $(As/Vm = F/m = N/V^2)$ being vacuum permittivity.

In the magnetic flux density $B = 1\ T$ the energy density is

$$w_B = \frac{1}{2}B^2/\mu_0 = \frac{1}{2}1^2/(4\pi \cdot 10^{-7}) = 0.398 \cdot 10^6\ J/m^3 = 0.111\ kWh/m^3\ , \tag{2}$$

$\mu_0 = 4\pi \cdot 10^{-7}\ Vs/Am$ $(Vs/Am = H/m = N/A^2)$ being vacuum permeability.

In the magnetic field of the Earth, for example, at $B = 45\ \mu T$, the energy density is

$$w_{ZB} = \frac{1}{2}(45 \cdot 10^{-6})^2/(4\pi \cdot 10^{-7}) = 0.806 \cdot 10^{-3}\ J/m^3 \tag{3}$$

Poynting vector indicates that most of the electromagnetic energy does not travel with an electric current inside an electrical conductor - example, an overhead power line - but in the space around the conductor: $\vec{S} = \vec{E} \times \vec{H}$ vectors, $\vec{S}$ the directed electromagnetic energy flux density, $\vec{E}$ the electric field strength, $\vec{H}$ the magnetic field strength.

2 The energy density in the gravitational field

How to calculate the energy density in the gravitational field - by analogy to the energy density in an electric field. Assumption: other disturbing electric charges and bodies are very distant:
Electric field strength at a distance r from the spherical charge Q:

$$E = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2} \tag{4}$$

voltage from the surface of a sphere with a radius r_0 to very distant regions:

$$U = \int_{r_0}^{\infty} E\ dr = -\frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r_0} \tag{5}$$

*Ul. Pavle Jeromnove 12, SI-1000 LJUBLJANA, Slovenia, mn.kozelj@gmail.com

element of volume, spherical shell: $dV = 4\pi r^2 \; dr$
and electrostatic energy of the entire area from the radius r_0 on:

$$W_E = \int_{r_0}^{\infty} \frac{1}{2} E^2 \epsilon_0 \; dV = \int_{r_0}^{\infty} \frac{Q^2}{8\pi\epsilon_0} \frac{dr}{r^2} = -\frac{Q^2}{8\pi\epsilon_0} \left(\frac{1}{r_0} - \frac{1}{r} \bigg|_{r\to\infty} \right) = -\frac{Q^2}{8\pi\epsilon_0} \frac{1}{r_0} \tag{6}$$

Hence

E_g, gravitation - analogously to the electric field strength E, etc.:
mutual acceleration force in the gravitational field, attractive forces, the signs abandoned

$$F = m_2 \frac{m_0 G_0}{r_{0-2}^2} = m_2 a_{0-2} \tag{7}$$

$G_o = 6.67408 \cdot 10^{-11} \; m^3 kg^{-1} s^{-2}$ ($m^3 kg^{-1} s^{-2} = Nm^2/kg^2$) being universal gravitational constant.
Gravitational field strength E_g at the distance r generated by a spherical mass m_0; analogously to ϵ_0 I introduce γ_0, and I am logically considering the previous equations (4) to (7) ... gravitational acceleration

$$E_g = \frac{m_0}{4\pi\gamma_0} \frac{1}{r^2} = m_0 \frac{G_0}{r^2} = a = a_g \qquad [m/s^2] \tag{8}$$

$\gamma_0 = \frac{1}{4\pi G_0}$
gravitational potential difference (gravitational "voltage")

$$U_g = \int_{r_0}^{\infty} E_g dr = \frac{1}{4\pi\gamma_0} \frac{m_0}{r_0} = G_0 \frac{m_0}{r_0} \qquad [m^2/s^2] \tag{9}$$

the gravitational energy in the space around the mass m_0 from the radius r_0 on

$$W_g = \int_{r_0}^{\infty} \frac{1}{2} E_g^2 \gamma_0 dV = \frac{m_0^2}{8\pi\gamma_0} \frac{1}{r_0} = \frac{1}{2} G_0 \frac{m_0^2}{r_0} = \Delta m_0 c_0^2 \qquad [J] \tag{10}$$

$c_0 = 2.997\,924\,58 \cdot 10^8 \; m/s$ being speed of light in vacuum
Δm_0 the amount of energy expressed in terms of mass deficit; specific:

$$\frac{\Delta m_0}{m_0} = \frac{1}{2} \frac{m_0}{r_0} \frac{G_0}{c_0^2} \tag{11}$$

and the density of the gravitational energy

$$w_g = \frac{1}{2} E_g^2 \gamma_0 = \frac{G_0}{8\pi} \frac{m_0^2}{r^4} = \frac{1}{2} a_g^2 \gamma_0 = \frac{a_g^2}{8\pi G_0} \qquad [J/m^3] \tag{12}$$

The gravitational energy inside the spherical mass:
if

$$m(r) = m_0 \frac{r^3}{r_0^3} \tag{13}$$

is a mass in a homogeneous sphere from the outer radius r_0 on to a radius r, the gravitational field strength at a radius r is

$$E_{gn} = m_0 G_0 \frac{r^3}{r_0^3} \frac{1}{r^2} = m_0 G_0 \frac{r}{r_0^3} \tag{14}$$

and the entire internal gravitational energy of the homogeneous spherical mass is

$$W_{gn} = \int_0^{r_0} \frac{1}{2} E_{gn}^2 \gamma_0 dV = \frac{1}{2} \frac{1}{4\pi G_0} (m_0 G_0)^2 \int_0^{r_0} \left(\frac{r}{r_0^3} \right)^2 (4\pi r^2) dr = \frac{1}{10} G_0 \frac{m_0^2}{r_0} = \frac{1}{5} W_{gz} \tag{15}$$

where $W_{gz} = W_g = \frac{1}{2} G_0 \frac{m_0^2}{r_0}$ the gravitational energy in the exterior outside of this sphere; if the sphere has homogeneous density inside its inside gravitational energy is $W_{gn} = \frac{1}{10} G_0 \frac{m_0^2}{r_0} = \frac{1}{5} W_{gz}$. If the density of the mass is arranged differently, $m = m(r)$, the factor for W_{gn} is different, is to be calculated by model of equations (8) to (15).

3 The gravitational energy at merging of masses/bodies in the Universe

For coefficient $k_1 > 0$ greater (or smaller) mass m_{01} connects with mass m_0 in the joint mass m_{02} bullet (sphere); simplified: all the specific densities of mass of masses before and after being merged are the same. Taking into account the geometry of the sphere it is obtained

$$\begin{gathered} m_{01} = k_1 m_0 \ , \quad m_{02} = m_0 + m_{01} = (1+k_1) m_0 \ , \\ r_{01} = r_0 k_1^{\frac{1}{3}} \ , \quad r_{02} = r_0 (1+k_1)^{\frac{1}{3}} \ , \quad k_1 > 0 \end{gathered} \tag{16}$$

$$W_{gz2} = \frac{1}{2} G_0 \frac{m_{02}^2}{r_{02}} = W_{gz0} \frac{(1+k_1)^2}{(1+k_1)^{\frac{1}{3}}} = W_{gz0} (1+k_1)^{\frac{5}{3}} \quad > W_{gz0} \tag{17}$$

$$\frac{\Delta m_{01}}{m_{01}} = \frac{1}{2} \frac{m_{01}}{r_{01}} \frac{G_0}{c_0^2} = \frac{1}{2} \frac{m_0}{r_0} \frac{k_1}{k_1^{\frac{1}{3}}} \frac{G_0}{c_0^2} = \frac{\Delta m_0}{m_0} k_1^{\frac{2}{3}} \tag{18}$$

$$\frac{\Delta m_{02}}{m_{02}} = \frac{1}{2} \frac{m_{02}}{r_{02}} \frac{G_0}{c_0^2} = \frac{1}{2} \frac{m_0}{r_0} \frac{(1+k_1)}{(1+k_1)^{\frac{1}{3}}} \frac{G_0}{c_0^2} = \frac{\Delta m_0}{m_0} \frac{(1+k_1)}{(1+k_1)^{\frac{1}{3}}} = \frac{\Delta m_0}{m_0} (1+k_1)^{\frac{2}{3}} \tag{19}$$

The ratio of the specific gravitational energy deficit of gravitational field of the new merged mass compared to the sum of the initial masses

$$\frac{\frac{\Delta m_{02}}{m_{02}}}{\frac{\Delta m_0}{m_0} + \frac{\Delta m_{01}}{m_{01}}} = \frac{(1+k_1)^{\frac{2}{3}}}{1 + k_1^{\frac{2}{3}}} \qquad < 1 \tag{20}$$

A mass deficit from the two bodies merging caused by the emission of the gravitational energy into space, is expressed as the sum of the mass deficits of the original bodies, as per (16) and (17):

$$W_{gz0} = \frac{1}{2} \frac{G_0 m_0^2}{r_0}, \quad W_{gz1} = W_{gz0} \cdot \frac{k_1^2}{k_1^{1/3}} = W_{gz0} k_1^{5/3}, \quad W_{gz2} = W_{gz0} (1+k_1)^{5/3} \tag{21}$$

$$K_{012} = \frac{W_{gz2}}{W_{gz0} + W_{gz1}} = \frac{(1+k_1)^{5/3}}{1 + k_1^{5/3}} > 1, \qquad \text{if } k_1 > 0 \tag{22}$$

Numerical examples for some ratios $k_1 = m_{01}/m_0$ are

$$\begin{aligned} & k_1 = 0 \ , && K_{012} = 1 \\ & k_1 = 0.1 \ , && K_{012} = (1+0.1)^{5/3}/(1+0.1^{5/3}) = 1.1474 \\ & k_1 = 1 \ , && K_{012} = 1.5874 \\ & k_1 = 2 \ , && K_{012} = 1.49474 \\ & k_1 = 0.5 \ , && K_{012} = 1.49474 \end{aligned} \tag{23}$$

and gravitational energy of merged mass is greater than sum of gravitational energies of the two bodies by factor of K_{012}, i.e.

$$W_{gz2} = K_{012} (W_{gz0} + W_{gz1}) \tag{24}$$

Gravitational energy (17) of the greater mass is obviously larger than that of the smaller mass (as long as the mass densities are unaffected), and is increasing very rapidly; specific mass density of greater masses are larger, which also means more gravitational energy; greater mass has more gravitational energy in the outside space as the individual components together; specific mass deficit (20) of coupled masses is less than the sum of consisting components - it is apparent, until the ratio $k_1 = \frac{m_{01}}{m_0}$ is not very high when m_0 nor comes to expression (does not influence anymore).

The bodies merging is increasing in its entirety the amount of mass deficit for the formation of a gravitational field of the new body in the Universe (22), (24), the bodies merging are emitting a gravitational wave.

The above equations and in next chapter calculated numerical amounts for W_g and w_g further highlight the results of the first measurements of gravitational waves.

These are the cases of the energy densities in the space of 1 m^3. Fields are the energy in the space.

Gravitational energy = own energy of gravitational field $\neq$ gravitational potential energy of an extraneous body between two points in the gravitational field of the first body, due to the force of gravity! - but there is a proportion.

4 In human living environment, on Earth

The energy densities in the air would be almost the same as in the vacuum, with the exception that molecules, atoms of gases and some other impurities are present.

In the gravitational field with acceleration of gravity, for example, $a_g = g_p = 9.83\ m/s^2$, the gravitational energy density is

$$w_g = \frac{g_p^2}{8\pi G_0} = \frac{9.83^2}{8\pi \cdot 6.67408 \cdot 10^{-11}} = 57.6 \cdot 10^9\ J/m^3 = 16.0\ MWh/m^3 \tag{25}$$

which means mass deficit $\Delta m_0 = 0.641 \cdot 10^{-6}\ kg/m^3$.

Most likely, for humanity it is a great luck nobody knows how to exploit this enormous gravitational energy - or it may became means of propulsion for interstellar travels in the (somewhat distant) future. But we know how to exploit energy of electric and magnetic fields, mostly in the large benefit for mankind.

The analogy, for example, is γ-radiation at the micro-level, which is, for example, the result of the events in the nucleus, where due to the change of internal field energy of the nucleus, and in the space around the nucleus respectively the atom (up "to the infinity"), and which is reflected as radiated quantum.

E.g., gravitational energy into the entire universe and the mass deficit of the external gravitational field of mass $m_0 = 5.97 \cdot 10^{24}\ kg$ and with the radius of $r_0 = 6.36 \cdot 10^6\ m$ are approximately

$$W_{gz} = 0.187 \cdot 10^{33}\ J \tag{26}$$

$$\Delta m_0 = 2.08 \cdot 10^{15}\ kg, \tag{27}$$

resp. relatively $\Delta m_0/m_0 = 0.349 \cdot 10^{-9}$.

5 Conclusion

The gravitational energy density in the vicinity of planets, stars and other bodies of large masses is enormous, also many orders of magnitude greater than the energy density of electric or magnetic fields. It is not known how to use it to the benefit of entire mankind. It is not clear what the consequences would be if the mankind would know and begin to exploit the gravitational field energy in large amounts, as has done and is doing with fossil fuels, nuclear energy and other resources.

When two large masses converge and merge (e.g., two neutron stars, "black hole swallows" a big space body), the gravitational fields change considerably, in the vicinity and in the entire universe. This pervasive changing of the gravitational field results in waves, detectable on the Earth with appropriate sensor systems. The above calculated numerical amounts for W_g and w_g further highlight the results of the first measurements of gravitational waves.

With the displacing masses in the space (in the Universe) there are also displaced their own gravitational fields. The fields in the space are changed, and the energy is transmitted through the space.

Fields are the energy in the space. These fields are intermediary of energy transmission through space, so trough the vacuum as through the matter, i.e., energy propulsion ("push off") throughout the space.

An indicator of the presence of the energy, or its gradient or its changes in discrete or distributed points of space and time, are in these points acting forces, or temperatures, which we know how to measure them in a variety of ways.